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Dualities, Twists, and Gauge Theories with Non-Constant Non-Commutativity

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Abstract

We study the world volume theory of D3-branes wrapping the Melvin universe supported by background NSNS B-field. In the appropriate decoupling limit, the open string dynamics is that of non-commutative guage field theory with non-constant non-commutativity. We identify this model as a simple Melvin twist of flat D3 branes. Along similar lines, one recognizes the model of Hashimoto and Sethi as being the Melvin null twist, and the model of Dolan and Nappi as being the null Melvin twist, of the flat D3-brane. This construction therefore offers a unified perspective on most of the known explicit constructions of non-commutative gauge theories as a decoupled theory of D-branes in a B-field background. We also describe the world volume theory on the D3-brane in Melvin universe which is decaying via the nucleation of monopole anti-monopole pair.

1 Introduction

Non-commutative gauge theory is an intriguing dynamical system which exhibits rich features such as gauge invariance, non-locality and UV/IR mixing. These are features commonly seen in more sophisticated theories such as string theory, little string theory, and gravity. Yet, they are simple enough to admit a Lagrangian formulation and for many purposes can be treated as an ordinary field theory. For this reason, they serve as useful toy model.

Non-commutative gauge theories have become the focus of intense investigation in the recent years in light of the realization that they arise naturally as a decoupling limit of open string dynamics in the presence of a background B-field [1–3]. So far, most of the work in this direction has focused on non-commutative gauge theories with constant non-commutativity parameter $\theta^{\mu\nu}$. Generalization of the Moyal $*$-product to general Poisson manifolds [4] and its relation to certain topological sigma model [5] are well known. Very interesting early work relating open string dynamics on weakly varying B field to non-constant non-commutativity parameter can be found in [6]. Nontheless, only a handful of concrete string theory realizations of gauge theories with non-constant non-commutativity are known [7–9].

In this article, we show that a large class of non-commutative gauge theories with non-constant non-commutativity can be constructed by applying a sequence of duality transformations and twists to flat D-branes. The prototype of this construction is a D-brane wrapping a Melvin universe supported by the flux of an NSNS B-field. We refer to such sequences of dualities and twists as *Melvin twists*. The sequence of steps is very similar to the ones used to construct dipole theories [10–13] but the orientation of the branes is different. Similar methods were used to construct supergravity solutions of Taub-NUT geometry in a non-trivial B-field background [14], as well as black branes [15] and Aichelburg-Sexel waves [16] in various asymptotic geometries. Slight variation in the construction of decoupled gauge theories can be characterized by the variation in the Melvin twists. In fact, we find that most of the known examples of non-commutative gauge theories with non-constant non-commutativity can be realized as a Melvin twist of a flat D-brane.

One interesting property of Melvin backgrounds in string theory is the fact that they are generically non-supersymmetric and can decay via nucleation of monopole anti-monopole pairs [17–20]. In light of the fact that the world volume theory of D-branes in a Melvin background is a non-commutative field theory, it is natural to expect that the world volume theory of D-branes in a decaying Melvin background would be some sort of non-commutative gauge theory with explicit time dependence. This scenario will also be explored in this paper.

The organization of this paper is as follows. We will begin in section 2 by formulating the decoupling limit of D-branes in a Melvin universe with background NSNS 3-form flux. This

example will serve as the prototype for the Melvin twist construction of non-commutative gauge theories. In section 3, we explain how the other explicitly known examples of non-commutative gauge theories, including the models of [7] and [8] can be viewed as a variant of the Melvin twist of flat D-branes. In section 4, we describe how the decay of Melvin space-time via nucleation of monopole anti-monopole pairs affects the world volume theory on the brane. We present our conclusions in section 5.

2 D-branes in Melvin universe

In this section we will describe the effective world volume dynamics of D-branes in a Melvin universe supported by NSNS 3-form field strength. As this example will serve as a prototype of the duality/twist construction of non-commutative gauge theories, we will describe the steps of the construction in some detail.

Let us begin by considering the closed string sector of the background. Melvin solutions supported by NSNS 3-form flux can be constructed by applying the following sequence of dualities and twists.

1. Start with a flat background in type IIB supergravity
$$ds^2 = -dt^2 + dr^2 + r^2 d\phi^2 + dz^2 + \sum_{i=1,6} dy_i^2 \tag{2.1}$$
where z is compactified on a circle with radius R.

2. T-dualize along z to obtain a background of type IIA supergravity
$$ds^2 = -dt^2 + dr^2 + r^2 d\phi^2 + d\tilde{z}^2 + \sum_{i=1,6} dy_i^2 \tag{2.2}$$
where the radius of $\tilde{z}$ coordinate is $\tilde{R} = \alpha'/R$.

3. This geometry admits an isometry generated by a vector $\partial/\partial\phi$. Given such an isometry vector, one can "twist" the compactification. By this, one means changing the Killing vector associated with the compactification from $(\partial/\partial\tilde{z})$ to $(\partial/\partial\tilde{z} + \eta\partial/\partial\phi)$. Alternatively, one can think of the twist as first replacing
$$d\phi \to d\phi + \eta\, d\tilde{z} \tag{2.3}$$
so that the metric reads
$$ds^2 = -dt^2 + dr^2 + r^2(d\phi + \eta d\tilde{z})^2 + d\tilde{z}^2 + \sum_{i=1,6} dy_i^2 \tag{2.4}$$
and then treating $\tilde{z}$ as the periodic variable of radius $\tilde{R}$ with ϕ fixed.

4. T-dualize along $\tilde{z}$ to obtain a solution of type IIB supergravity

$$\begin{aligned} ds^2 &= -dt^2 + dr^2 + \frac{r^2}{1+\eta^2 r^2} d\phi^2 + \frac{1}{1+\eta^2 r^2} dz^2 + \sum_{i=1}^{6} dy_i^2 \\ B &= \frac{\eta r^2}{1+\eta^2 r^2} d\phi \wedge dz \\ e^{\phi} &= \sqrt{\frac{1}{1+\eta^2 r^2}} \,. \end{aligned} \tag{2.5}$$

This solution describes a Melvin universe supported by a background NSNS 3-form flux. Our objective now is to consider the world volume theory of D-branes in this background. There is some freedom in the choice of embedding of D-branes in this background. In fact, precisely this sequence of duality transformation in the presence of a D-brane was used in the construction of dipole theories in [10,11]. In the context of dipole theories, the D3-brane was embedded in such a way that it is extended along the (t, z, y_1, y_2) directions, but localized in the (r, ϕ) plane. In other words, the isometry along which we twisted the background corresponds to the R-symmetry from the point of view of the D-brane world volume theory. In the appropriate scaling limit, the theory on the world volume becomes a theory of dipoles whose length is proportional to the R-charge.

Alternatively, one can embed a D3-brane in such a way that it is extended along the (t, r, ϕ, z) and localized in the y_i directions. This time, the isometry with which we twist the compactification is associated to the rotation along the world volume of the gauge theory. One therefore expects to find a non-local theory whose degree of non-locality is proportional to the angular momentum quantum number.

In order to read off the parameters appropriate for interpreting the dynamics from the open string point of view, we apply the mapping of Seiberg and Witten [3]

$$(G + \frac{\theta}{2\pi\alpha'})^{\mu\nu} = [(g+B)_{\mu\nu}]^{-1} \,. \tag{2.6}$$

When this formula is applied to the closed string background (2.5), one finds

$$\begin{aligned} G_{\mu\nu} dx^\mu dx^\nu &= -dt^2 + dr^2 + r^2 d\phi^2 + dz^2 \\ \theta^{\phi z} &= 2\pi\alpha'\eta \end{aligned} \tag{2.7}$$

Therefore, in order to extract a field theory limit keeping the effect of non-locality finite, one should scale η so that

$$2\pi\alpha'\eta = 2\pi\Delta = \text{finite} \tag{2.8}$$

while sending $\alpha' \to 0$. We also keep the radius R of the periodic z coordinate finite in this limit. In terms of the Cartesian coordinates we have

$$\theta^{x_1 z} = -\theta^{z x_1} = -2\pi\Delta x_2, \qquad \theta^{x_2 z} = -\theta^{z x_2} = 2\pi\Delta x_1 \tag{2.9}$$

with all other components vanishing. This is an example of non-commutative gauge theory with non-constant non-commutativity for which one needs to employ the formula of Kontsevich [4] to define the appropriate $*$-product. One can readily verify that the condition

$$\theta^{il}\partial_l\theta^{jk} + \theta^{jl}\partial_l\theta^{ki} + \theta^{kl}\partial_l\theta^{ij} = 0 \tag{2.10}$$

which is necessary for associativity, is satisfied by (2.9).

The formula (2.6) of Seiberg and Witten was originally derived for the case of a constant B-field background. One can nonetheless see that the open string metric properly captures the effective dynamics of open strings by observing that the induced metric (2.4) on the D2-brane after T-dualizing along the z direction is identical to the open string metric found in (2.7). The T-dual system consists of an array of type IIA D2-branes at fixed positions in the periodic $\tilde{z}$ coordinate in the background space-time (2.4). Open strings ending on these D2-branes will roughly follow geodesics along the t-r-ϕ plane, which we see is identical to the geodesic of the open string metric (2.7). Further, T-duality along the z direction does not affect the general features of the motion of these strings along the t-r-ϕ coordinates.

One can also reach similar conclusions by observing that the Polyakov action

$$S = \frac{1}{4\pi\alpha'}\int d\tau\, d\sigma \left(\sqrt{-\gamma}\gamma^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu g_{\mu\nu} - \epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu B_{\mu\nu}\right) \tag{2.11}$$

for open strings parameterized by strip coordinates $-\infty < \tau < \infty$ and $0 < \sigma < \pi$, in the background (2.5), admits a solution of the equation of motion, constraints, and the boundary condition of the form

$$\begin{aligned}
t(\tau,\sigma) &= \left(\frac{L}{\pi\eta b}\right)\tau \\
\rho(\tau,\sigma) &= \sqrt{b^2 + t(\tau,\sigma)^2} \\
\phi(\tau,\sigma) &= \tan^{-1}\left(\frac{t(\tau,\sigma)}{b}\right) \\
z(\tau,\sigma) &= \frac{L}{\pi}\sigma \\
y_i(\tau,\sigma) &= y_i \\
\gamma_{\alpha\beta}(\tau,\sigma) &= \eta_{\alpha\beta}\ .
\end{aligned} \tag{2.12}$$

This solution describes the motion of an open string which is extended like a rod along the z direction. The string is traveling on a straight line in the r-ϕ plane with impact parameter b and is propagating at the speed of light with respect to the open string metric. This is further evidence supporting the relevance of $G_{\mu\nu}$ for the effective dynamics of open strings. The angular momentum of this configuration is

$$J = \int d\sigma \frac{\partial\mathcal{L}}{\partial(\partial_\tau\phi)} = \frac{L}{2\pi\alpha'\eta}\ . \tag{2.13}$$

Combining this result with (2.7), one can easily verify that the length of the rod is is proportional to angular momentum

$$L = \theta^{\phi z} J \tag{2.14}$$

with the non-commutativity parameter being the constant of proportionality. The fact that the open strings become dipoles of length L gives rise to non-locality in their interaction, precisely of the type that one would expect for the non-commutativity between angular coordinate ϕ and a Cartesian coordinate z.

String theory in space-times generated by acting with Melvin twists are implicitly simple. It is possible to quantize the strings and to construct explicit boundary states describing D-branes in these backgrounds [21, 22]. One can also take advantage of intrinsic simplicity to derive the supergravity dual of the non-commutative gauge theory which arise as a decoupling limit of D3-brane in (2.5). Simply start with the supergravity solution of the D3-brane

$$\begin{aligned} ds^2 &= f(\rho)^{-1/2}(-dt^2 + dr^2 + r^2 d\phi^2 + dz^2) + f(\rho)^{1/2}(d\rho^2 + \rho^2 d\Omega_5^2), \\ f(\rho) &= 1 + \frac{4\pi g N \alpha'^2}{\rho^4} \end{aligned} \tag{2.15}$$

and follow the chain of dualities outlined in this section. Finally, scale

$$\rho = \alpha' U, \qquad \eta = \frac{\Delta}{\alpha'} \tag{2.16}$$

and send $\alpha' \to 0$ keeping U and Δ fixed. The resulting geometry is given by a metric which in string frame has the form

$$ds^2 = \alpha' \left(\frac{U^2}{\sqrt{\lambda}} \left(-dt^2 + dr^2 + \frac{r^2 d\phi^2 + dz^2}{1 + \frac{\Delta^2 r^2 U^2}{\lambda}} \right) + \frac{\sqrt{\lambda}}{U^2}(dU^2 + U^2 d\Omega_5^2) \right) . \tag{2.17}$$

This is essentially the construction used in [7, 11] with minor difference in the orientation of the brane and the isometry of the twist. The resulting space-time is the supergravity dual of the non-commutative gauge theory with non-commutativity (2.9) along the lines of [23, 24].

3 Other Melvin twists and non-commutative gauge theories

In the previous section, we described the construction of non-commutative gauge theory with the non-constant non-commutativity indicated in (2.9). Essentially, this is a simpler version of the construction of a theory with time dependent non-commutativity parameter outlined in [7]. In this section, we will examine if other non-commutative gauge theories admit a realization as the world volume dynamics of D-branes in a closed string background which is a Melvin twist of flat space.

One non-commutative gauge theory which has been studied by Dolan and Nappi [8] is the world volume theory of D-branes in the Nappi-Witten background [25]. The Nappi-Witten background can be viewed as a 3+1 dimensional plane wave supported by a null background NSNS 3-form flux. It can be generated by performing the Melvin twist along the light-like direction, or equivalently, by combining the chain of dualities leading to the construction of the Melvin universe (2.5) with a boost. Following [15] we refer to this sequence of solution generating transformations as the *null Melvin twist*. Detailed explanation of steps and scalings involved with the null Melvin twist can be found in [13, 15]. The Null Melvin twist applied to flat the background of type IIB supergravity gives rise to a solution

$$\begin{aligned} ds^2 &= -dt^2 + dz^2 - 2\beta^2 r^2 (dt+dz)^2 + dr^2 + r^2 d\phi^2 + \sum_{i=1}^{6} dy_i^2 \\ B &= \beta r^2 d\phi \wedge (dt+dz) \\ e^{\phi} &= 1 \ . \end{aligned} \tag{3.1}$$

Now consider a D3-brane extended along (t, r, ϕ, z) and localized along the y_i directions. It is not difficult apply the Seiberg-Witten map (2.6) to show that the open string metric and the non-commutativity parameter are given by

$$G_{\mu\nu} dx^\mu dx^\nu = -dt^2 + dr^2 + r^2 d\phi^2 + dz^2, \qquad -\theta^{\phi t} = +\theta^{t\phi} = \theta^{\phi z} = -\theta^{z\phi} = 2\pi\alpha'\beta \ . \tag{3.2}$$

One can therefore obtain a non-commutative gauge theory with finite non-commutativity parameter by taking the limit $\alpha' \to 0$ keeping

$$\Delta = \alpha'\beta = \text{fixed} \ . \tag{3.3}$$

In Cartesian coordinates, we have

$$-\theta^{x_1 t} = \theta^{t x_1} = \theta^{x_1 z} = -\theta^{z x_1} = -2\pi\Delta x_2, \qquad -\theta^{x_2 t} = \theta^{t x_2} = \theta^{x_2 z} = -\theta^{z x_2} = 2\pi\Delta x_1 \ . \tag{3.4}$$

This non-commutativity is non-constant, but not time dependent. This is different from t dependent non-commutativity which was reported in [8]. This difference can be attributed to the difference in the coordinates and the gauge for the NSNS 2-form of the closed string background. To see this more explicitly, note that the Nappi-Witten background used in [8] used the coordinate

$$\phi = \phi' + \beta(t+z) \tag{3.5}$$

so that the metric takes the form

$$ds^2 = -dt^2 + dz^2 - 2\beta r^2 (dt+dz) d\phi' + dr^2 + r^2 d\phi'^2 + \sum_{i=1}^{6} dy_i^2 \tag{3.6}$$

while the B field differs from (3.1) by a total derivative term

$$B' = \beta r^2 d\phi' \wedge (dt + dz) + d(\beta r^2 (t+z) d\phi') = 2\beta r (t+z)\, dr \wedge d\phi' \ . \tag{3.7}$$

In this gauge, the B' field has explicit dependence on t which is suggestive of a time dependent non-commutativity.

However the equation of motion for the gauge field derived from the DBI action for a general closed string background [26] does not admit $F = 0$ as a solution for (3.7). So (3.7) with trivial gauge field $F = 0$ is not a consistent background of string theory. On the other hand, (3.1) does admit $F = 0$ as a consistent solution to the equation of motion [27]. We are therefore led to conclude that the open string metric and the non-commutativity parameter in the decoupling limit of D3-branes in Nappi-Witten background are that of (3.2).

It is interesting to note that other known examples of non-commutative gauge theory can be viewed as being generated by a sequence of Melvin-like twists. For example, the standard non-commutative gauge theory with constant space-like non-commutativity can be thought of as being generated via a sequence where one

1. Start with flat space
$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2 + d\rho^2 + \rho^2 d\Omega_5^2 \tag{3.8}$$
2. T-dualize the compact coordinate z with radius R so that it becomes a compact coordinate $\tilde{z}$ with radius α'/R.
3. Twist by replacing $dy \to dy + (\Delta^2/\alpha')d\tilde{z}$
4. T-dualize back on $\tilde{z}$.

This will give rise to a constant NSNS B-field which via the Seiberg-Witten map gives rise to a theory with non-commutativity $\theta^{yz} = 2\pi\Delta^2$. Because this amounts to twisting along the isometry $\partial/\partial y$ which corresponds to shift in the y direction, one might refer this sequence of dualities as *Melvin shift twist*.

Combining the Melvin shift twist with a boost along the z coordinate will give rise to a non-commutative gauge theory with light-like non-commutativity [28]. Since this is equivalent to T-dualizing along the null direction accompanied by a twist by shift along the y coordinate, one might refer to this construction as the *null Melvin shift twist*.

One might also consider combining T-duality along the z direction and a twist with respect to boost along the x coordinate. This will give rise to a background which is T-dual

Type of Twist	Model
Melvin Twist	Model of section 2
Null Melvin Twist	Dolan-Nappi model
Melvin Shift Twist	Moyal model
Null Melvin Shift Twist	Aharony-Gomis-Mehen model
Melvin Null Twist	Hashimoto-Sethi model
Melvin R Twist	Bergman-Ganor model
Null Melvin R Twist	Ganor-Varadarajan model

Table 1: Catalog of non-commutative gauge theories viewed as a world volume theory of D-branes in a "X" Melvin "Y" twist background.

to the background considered in [29]. However, the fact that $\tilde{z}$ coordinate is time-like in some region gives rise to pathology which makes interpretation of the decoupled theory as a non-commutative theory unreliable. This can be cured by twisting by a combination of a boost and a rotation so that the net effect is twisting by a null rotation. An appropriate name for this sequence of dualities and twists is the *Melvin null twist*. This gives rise to a closed string background which is a T-dual of the null brane [30, 31]. In the presence of D3-brane, this gives the construction of [7]. (The S-dual corresponding to NCOS with time dependent non-commutativity was considered in [32].)

The non-commutative gauge theories generated along these lines are summarized in table 1. It is interesting to observe that most of the known examples of non-commutative gauge theories can be thought of as being generated by sequences of dualities and twists. We have also included twists along the direction transverse to the brane which we refer to as the R-twist.

A notable omission in this catalog is the model of Lowe-Nastase-Ramgoolam [9]. From the point of view of constructing a non-commutative gauge theory, it is convenient to think of this setup as placing a D3-brane probe in the background of smeared NS5-branes. While this is a perfectly sensible closed string background that one can consider, it appears not to be related to flat space under any chain of dualities.

4 World volume theory of branes in a decaying Melvin background

Each of the non-commutative gauge theories enumerated in Table 1 are interesting in their own right. The example of embedding D3-branes in a Melvin universe, however, is special in that the closed string background breaks supersymmetry and is susceptible to decay.

There are several decay modes which arise as double wick rotation of black holes solutions, originally considered in [17–20] and more recently in the context of string theory in [33]. It would be interesting to see what kind of effective open string dynamics arise as a decoupling limit of D-branes placed in this class of time dependent background. It is natural to expect that the world volume theory will inherit the time dependence of the background space-time, giving rise to a new type of non-commutative gauge theory with explicit time dependence.

Since we are interested in the interplay between background NSNS 3-form flux and the non-commutativity on the world volume of D3-branes, it is natural to consider the decay mode via nucleation of monopole anti-monopole pair which subsequently fly off to infinity. To be more concrete, we consider a background where a monopole and an anti-monopole, which are smeared along $y_2 \ldots y_6$ coordinates, undergoes uniform acceleration along the y_1 direction. This is precisely the process which was considered in detail in [34]. Our interest is in exploring the effect of the accelerating monopole anti-monopole pair on the world volume theory of D3-brane extended along (t, r, ϕ, z) coordinates and localized half way between the monopole anti-monopole at $y_1 = 0$.

Let us review the decaying Melvin solution more explicitly. The most efficient way to describe this background is to start from the Euclidean Kerr instanton solution in 5 dimensions

$$\begin{aligned} ds^2 &= r^2 \cos^2\theta\, d\psi^2 + (r^2 - \alpha^2 \cos\theta^2)\, d\theta^2 + \frac{r^2 - \alpha^2 \cos\theta^2}{r^2 - \alpha^2 - \mu}\, dr^2 \\ &\quad + dz^2 + \sin^2\theta (r^2 - \alpha^2) d\phi^2 - \frac{\mu}{r^2 - \alpha^2 \cos^2\theta}(dz + \alpha \sin^2\theta d\phi)^2 \end{aligned} \tag{4.1}$$

and do a Wick rotation $\psi = it$

$$\begin{aligned} ds^2 &= -r^2 \cos^2\theta\, dt^2 + (r^2 - \alpha^2 \cos\theta^2)\, d\theta^2 + \frac{r^2 - \alpha^2 \cos\theta^2}{r^2 - \alpha^2 - \mu}\, dr^2 \\ &\quad + dz^2 + \sin^2\theta (r^2 - \alpha^2) d\phi^2 - \frac{\mu}{r^2 - \alpha^2 \cos^2\theta}(dz + \alpha \sin^2\theta d\phi)^2 \ . \end{aligned} \tag{4.2}$$

This geometry covers the (t, r, ϕ, z, y_1) plane in different coordinates. We would like to consider the z coordinate to be compact with period $2\pi R$. The Euclidean horizon velocity of this metric is

$$\Omega = -\frac{g_{\phi z}}{g_{\phi\phi}} = \frac{\alpha}{\mu} \ . \tag{4.3}$$

It is therefore natural to compactify along the Killing vector $(d/dz) + \Omega(d/d\phi)$. One can equivalently twist the coordinates

$$z = \tilde{z}, \qquad \phi = \tilde{\phi} + \Omega \tilde{z} \tag{4.4}$$

and compactify along $(d/d\tilde{z})$. Generically, there will be a conical singularity in the r-$\tilde{z}$ plane, unless the period of the compactification is adjusted according to the parameter of the Kerr

instanton solution. This is achieved by choosing the radius of compactification according to

$$\tilde{z} = \tilde{z} + R, \qquad R^2 = \frac{\mu^2}{\mu + \alpha^2} \ . \tag{4.5}$$

This is the analogue of the "bubble of nothing" for the Melvin universes [35]. One can actually consider more general twists

$$z = \tilde{z}, \qquad \phi = \tilde{\phi} + \left(\Omega + \frac{n}{R}\right)\tilde{z} \ . \tag{4.6}$$

The case $n = 1$ is what is referred to as the "Shifted Kerr" solution. This is the solution which corresponds to the Ernst solution in different coordinates as was shown in [19]. Since the original Kerr instanton is asymptotically flat, its Lorentzian continuation is also flat at large spatial distances away from the origin. The dimensional reduction of this 4+1 dimensional geometry along a twisted z coordinate will therefore at large distances give rise to a Melvin universe supported by the flux of the Kaluza-Klein gauge field. The presence of the Kerr instanton will manifest itself as a Kaluza-Klein monopole anti-monopole pair flying away in this background Melvin geometry. The twist parameter η for this geometry at spatial infinity is

$$\eta = \frac{\alpha}{\mu} + \frac{1}{R} \ . \tag{4.7}$$

This 4+1 dimensional geometry can be embedded into type IIA or type IIB supergravity in 10 dimension by simply adjoining 5 flat directions. We will consider the case of IIA where the Kaluza-Klein monopole is now a five-brane. T-dualizing this geometry along $\tilde{z}$ will give rise to an asymptotically Melvin solution supported by NSNS B-field in IIB supergravity. This is the same Melvin geometry considered in section 2. The IIA Kaluza-Klein monopole becomes NSNS 5-brane extended along the y_2-y_6 directions and smeared along the $\tilde{z}$ direction in this picture.

Roughly speaking, the monopole anti-monopole pair partially screens the Melvin flux. The natural place to probe this effect is along the plane which is half way between the monopole and the anti-monopole. Unfortunately, the coordinates used in writing (4.2) is not adequate for this purpose for it does not cover the entire geometry and misses this important region. The fact that the coordinates of (4.2) is incomplete can be seen by noting that for large r, t-θ-ϕ plane describes de Sitter space in static coordinates, which is geodesically incomplete. This problem is easy to rectify. Simply go to the global de Sitter coordinates by mapping

$$t = \sinh^{-1}\left(\frac{\sinh\tilde{t}}{\sqrt{1-\cosh^2\tilde{t}\sin^2\tilde{\theta}}}\right), \qquad \theta = \sin^{-1}(\cosh\tilde{t}\sin\tilde{\theta}) \ . \tag{4.8}$$

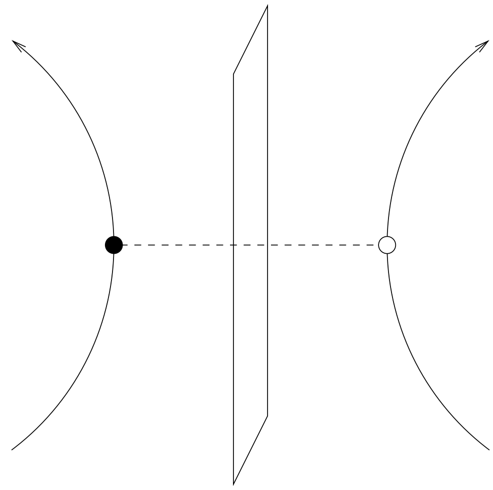

Figure 1: Configuration of D3-brane oriented along the plane bisecting the monopole anti-monopole pair in a decaying Melvin universe

Now, the plane bisecting the monopole anti-monopole pair can be identified as the surface $\tilde{\theta} = \pi/2$.

At this point one can T-dualize along $\tilde{z}$, place the D3-brane at $\tilde{\theta} = \pi/2$, and study the effective open string dynamics. It can be checked that the ansatz $\tilde{\theta} = 0$, $y_i = 0$ and vanishing world volume gauge field is consistent with the equation of motion of the DBI action in this background. In order to take the decoupling limit, we will scale

$$\eta = \frac{\Delta}{\alpha'}, \qquad R = \frac{2\pi\alpha'}{L} \tag{4.9}$$

for fixed Δ and L as we send α' to zero. The scaling of R is necessary in order for the z coordinate in the IIB picture to have finite period $z \sim z + L$. This will cause μ and α^2 to scale like α'^2.

Our task now is to extract the open string metric and the non-commutativity parameter in this scaling limit. As we found before, naive application of Seiberg-Witten formula (2.6) gives the same metric in the $\tilde{t}$-r-ϕ plane as the IIA metric in the T-dual picture, ensuring that the open string metric properly captures the geodesic motion of the open strings.

Along the $\tilde{t}$-r coordinates, the induced metric on the D3-brane takes the form

$$(r^2 + \alpha^2 \sinh^2 \tilde{t}) \left(-d\tilde{t}^2 + \frac{dr^2}{r^2 - \mu - \alpha^2} \right) . \tag{4.10}$$

At large r, this looks like a Rindler coordinate. However, this coordinate ends at $r^2 = \mu + \alpha^2$ and the Rindler horizon is not encountered. Instead, the region near $r^2 = \mu + \alpha^2$ is actually regular. To exhibit this feature, it is useful to go to the null coordinates

$$u = t + r_*, \qquad v = t - r_* \tag{4.11}$$

where

$$r_* = \int dr \sqrt{\frac{g_{rr}}{g_{tt}}} = \log\left(\frac{r+\sqrt{r^2-\mu-\alpha^2}}{\sqrt{\mu+\alpha^2}}\right) , \tag{4.12}$$

and make the standard transformation to take the Rindler coordinate into Cartesian coordinates

$$u = \log\left(\frac{U}{\sqrt{\mu+\alpha^2}}\right), \qquad v = -\log\left(-\frac{V}{\sqrt{\mu+\alpha^2}}\right) . \tag{4.13}$$

The cut-off $r^2 = \mu + \alpha^2$ now corresponds to

$$UV = -(\mu + \alpha^2) \tag{4.14}$$

which is a time-like curve. This time-like curve can be mapped to curve $x = 0$ parameterized by τ by making further coordinate transformations

$$U = \tilde{U} + \sqrt{\tilde{U}^2 + \mu + \alpha^2}, \qquad V = \tilde{V} - \sqrt{\tilde{V}^2 + \mu + \alpha^2} \tag{4.15}$$

and

$$\tilde{U} = \tau + x, \qquad \tilde{V} = \tau - x . \tag{4.16}$$

In these coordinates, the closed string metric in the neighborhood of $x = 0$ looks like

$$ds^2 = \left(\frac{\alpha^4 + \mu^2 + \alpha^2\ (2\,\mu + \tau^2)}{(\alpha^2+\mu)\ (\alpha^2+\mu+\tau^2)}\right)\left(-d\tau^2 + dx^2 + x^2 d\phi^2 + dz^2\right) \tag{4.17}$$

which is perfectly regular.

Now we are ready to compute the open string parameters. The scaling (4.9) dictates that

$$q^2 \equiv \frac{\alpha^2}{\mu} = \frac{(L-2\pi\Delta)^2}{4\pi\Delta(L-\pi\Delta)} \tag{4.18}$$

is fixed and in the decoupling limit, the open string metric develops a discontinuity: we find

$$ds^2_{open} = \left(1 - \frac{\Theta(\tau^2 - x^2)}{1+q^2}\right)(-d\tau^2 + dx^2) + x^2 d\phi^2 + dz^2 \tag{4.19}$$

where $\Theta(x)$ is the step function. The non-commutativity parameter comes out to

$$\theta^{\phi z} = 2\pi\Delta . \tag{4.20}$$

This appears to describe a non-commutative gauge theory with additional explicit time dependence in the form of the metric. The geometry is almost flat. For $x^2 - \tau^2 > 0$, it is identical to the non-commutative gauge theory of section 2. Perhaps this is to be expected since in the decoupling limit, both μ and α is going to zero.

5 Conclusions

In this article, we examined the world volume theory of D3-branes wrapping a Melvin universe supported by NSNS B-field and found that it describes a non-commutative gauge theory with non-constant non-commutativity in the appropriate scaling limit. Melvin universes are particularly simple in that they can be gotten from applying dualities and twists on flat space. The particular sequence of transformations leading to the Melvin universe is called the Melvin twist. We find that many examples of non-commutative guage theories with non-constant non-commutativities can be generated from a slight variation of the Melvin twist. The model of Hashimoto and Sethi for example arises from Melvin null twist, whereas the model of Dolan and Nappi can be realized as the null Melvin twist. Melvin twists appear to provide a unified perspective on most of the explicitly known construction of non-commutative gauge theory as a decoupled theory on branes.

We also studied the world volume theory of D-brane embedded into Melvin universe decaying via nucleation of monopole anti-monopole pair. Such a background exhibits rich dynamics in the closed string sector and was investigated in detail in [34]. Unfortunately, most of the time dependence appear to be smoothed out in the process of taking the decoupling limit of the world volume gauge theory. The only remnant of the time dependence we find is a discontinuity in the open string metric along the light-cone. Perhaps the physics of open strings where one does not completely decouple the closed strings and excited states will exhibit more interesting dynamics.

Models constructed using Melvin twists are particularly simple. The world sheet theory for strings in these background are exactly solvable and so it should be possible to make many precise statements about these theories from the string theory point of view. It is also possible to write down an explicit action for these non-commutative gauge theories and analyze perturbative issues as was done in [36, 37]. It would also be interesting to explore the instanton, monopole, and vortex solutions of non-commutative gauge theories [38–45], and to study the structure of gauge invariant operators [46, 47], along the lines of what was done for the theories with constant non-commutativity parameters.

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